Basel II IRB Risk Weight Functions

Demonstration and Analysis

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By

Benoit Genest & Leonard Brie

Global Research & Analytics¹

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Research Team. Tel.: +33-(0)1-80 18 26 18; fax: +33-(0)1-80 18 26 20
E-mail :bgenest@chappuishalder.com (London)
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Abstract

Regulatory capital requirements pose a major challenge for financial institutions today.

As the Asian financial crisis of 1997 and rapid development of credit risk management revealed many shortcomings and loopholes in measuring capital charges under Basel I, Basel II was issued in 2004 with the sole intent of improving international convergence of capital measurement and capital standards.

This paper introduces Basel II, the construction of risk weight functions and their limits in two sections:

In the first, basic fundamentals are presented to better understand these prerequisites: the likelihood of losses, expected and unexpected loss, Value at Risk, and regulatory capital. Then we discuss the founding principles of the regulatory formula for risk weight functions and how it works.

The latter section is dedicated to studying the different parameters of risk weight functions, in order to discuss their limits, modifications and impacts on the regulatory capital charge coefficient.

Key words: Basel II, EL, UL, LGD, PD, Merton’s Model, ASRF, Conditional PD, rho, Maturity, confidence level, PDF.

JEL Classification: C1, G21
1. Founding Principles of IRB risk weight functions

This section covers the basic fundamentals of loss and economic capital in order to thoroughly explain the founding principles of the IRB regulatory curves.

1.1. Basics fundamentals

**Likelihood of losses and required economic capital**

We begin with a walkthrough of the basic components in the diagram above: *expected loss*, *unexpected loss*, *Value at Risk* and *Required Economic Capital* (Regulatory Capital).

There is no doubt that it is impossible to forecast the exact amount of potential losses a bank will suffer in any given year. In fact, banks can only forecast the average level of those credit losses, which are called *expected losses* and denoted by *(EL)*. In addition, banks are required to cover these *expected losses (EL)* with accounting provisions.

*Unexpected losses (UL)* are losses above expected levels that banks expect to incur in future, but cannot predict their timing or severity.

Although high levels of interest are charged on credit exposures, a bank cannot absorb all of its unexpected losses. Therefore, these losses must be covered by *Economic Capital (EC)*
which has a loss-absorbing function. After all, “economic capital” protects a bank the same way a shield protects a fighter, by defending against unexpected losses.

With regards to the Value at Risk (VaR), it’s nothing more than the maximum potential loss with a confidence level of a percentage $\alpha$. This loss would be exceeded by only a very small probability of $1-\alpha$.

**Formula:**

\[
\begin{align*}
EL &= PD \times EAD \times LGD \\
UL &= VaR(\alpha) - EL \\
EC &= UL
\end{align*}
\]

**Where:**

- $PD$: Probability of Default
- $EAD$: Exposure at Default
- $LGD$: Loss given Default

**Regulatory Capital Charge measurement approach**

Regulatory capital charge can be measured using the standard or Internal Rating Based (IRB) approach, however there are major differences between the two with respect to the risk weights used.

In the standard approach, the capital charge formula is:

**Capital Charge** = $EAD \times \text{regulatory risk weight coefficient} \times 8\%$

The risk weighting coefficient is based on the quality of the loan quantified by external ratings. However, for some institutions like the BIS, IMF, ECB, EC and MDBs, the risk weight is always 0%. That means these institutions are considered solvent at all times by BCBS.
For sovereign banks and corporations, the risk weights are represented in the table below:

<table>
<thead>
<tr>
<th>Rating</th>
<th>Sovereigns</th>
<th>Banks</th>
<th>Corporates</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA – AA-</td>
<td>0%</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>A+ – A-</td>
<td>20%</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>BBB+ – BBB-</td>
<td>50%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>BB+ – BB-</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>B+ – B-</td>
<td>100%</td>
<td>100%</td>
<td>150%</td>
</tr>
<tr>
<td>Below B-</td>
<td>150%</td>
<td>150%</td>
<td>150%</td>
</tr>
<tr>
<td>Unrated</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Source: Bank for International Settlements

In the IRB approach, the weighting coefficient is based on internal ratings. Instead of relying on an outside rating agency, banks are required to estimate their own rating “in-house” by effectively using internal rating systems.

In this framework, the regulatory formula for capital charge is not the same as the one in the standard approach. It is as follows:

\[
\text{Capital charge} = EAD \times \text{regulatory risk weight function} \{PD; LGD; M; \rho\} \times 8\%
\]

Indeed, the risk weight in the IRB approach is a function depending on PD, LGD, M (Maturity) and \( \rho \) (correlation coefficient) and not a fixed coefficient depending only on an external rating.

Furthermore, there are two types of IRB approaches: Foundation IRB and Advanced IRB.

In Foundation IRB:

- Banks use their own estimates of PD.
- The LGD is fixed at 45% for senior claims on corporates, sovereigns and banks not secured by recognized collateral.
• The LGD is fixed at 75% for all subordinated claims on corporates, sovereigns and banks.
• The effective maturity (M) is fixed at 2.5 years for corporate exposures except for repo-style transactions where the maturity is fixed at 6 months.

In Advanced IRB:
• The banks use their own estimates of both PD and LGD.
• The effective maturity is not fixed and banks use their own estimates for each exposure.

It’s worth mentioning that banks have to respect minimum guidelines to be authorized to use an IRB approach.

An IRB approach is better to use than the standard if the regulatory weighting function calculates a risk weight below the one fixed in the standard approach. As an example, for corporates, if we found a risk weight below 100%, the IRB approach would allow us to have a less restrictive capital charge and hence free up resources that can be directed to profitable investments.

In addition, using an Advanced IRB approach could be more advantageous because of high LGDs imposed in the foundation of the approach. This could encourage banks to invest in developing LGD models in order to get regulatory approval.

1.2. From Merton’s model to the risk weight functions

The regulatory formula of risk weight functions has been obtained on the basis of simple assumptions from Merton’s model (also known as "Model of the Firm").
The figure below is presented to better understand Merton’s Model.

---

2 According to the BCBS, a subordinated loan is a facility that is expressly subordinated to another facility.
Merton’s Model postulates that the counterparty defaults when the value of its assets is less than that of its debts. To be more precise, it’s a firm’s distance to default, which calculates the difference between the value of assets and debts of a given firm. The larger the distance to default, the more solvent the firm is.

The Basel Committee has adopted this principle, adding significant assumptions such as the infinite granularity of considered portfolios, which means the contribution of an individual exposure to the portfolio is insignificant and that correlation between assets is not taken into account. Also, a time horizon of 1 year is assumed by the Basel committee. As a result of these modifications, the regulatory formula became known as the Asymptotic Single Risk Factor (ASRF) model.

The ASRF model is used to define the asset value. This model postulates that the value of an asset depends on two factors:

- The systematic factor which models the global environment, generally the overall state of the economy
- Idiosyncratic factor which is specific to the asset

Formally, the ASRF model could be presented as follows:

\[ R_t = \sqrt{\rho_i}Y + \sqrt{(1 - \rho_i)}\varepsilon_i \]

*With:*

- \( R_t \): Value of the asset
- \( \rho_i \): Correlation coefficient
We define the “default” variable as a binomial variable $Z_i$ with the following distribution

$$Z_i = \begin{cases} 
1 & \text{with the probability } p_i \\
0 & \text{with the probability } 1 - p_i
\end{cases}$$

Which means that an asset $i$ default with the probability $p_i$.

In the framework of Merton’s model, it is assumed that an asset $i$ default if it’s value goes below a critical threshold $S_i$.

Formally we can write:

$$Z_i = 1 \ iif \ R_i \leq S_i \quad (1)$$

From which we deduce:

$$\mathbb{P}(Z_i = 1) = \mathbb{P}(R_i \leq S_i)$$

Knowing the $Z_i$ distribution this is equivalent to:

$$p_i = \mathbb{P}(R_i \leq S_i)$$

Since $R_i \sim N(0,1)$ we have:

$$p_i = \Phi(S_i)$$

Where $\Phi(\cdot)$ is the cumulative normal distribution, therefore:

$$S_i = \Phi^{-1}(p_i)$$

Thus from formula (1) above we deduce:

$$Z_i = 1 \ iif \ R_i \leq \Phi^{-1}(p_i)$$

As a result, the critical threshold is nothing else than a function of the default probability of an asset.
The value of an asset $R_i$ depends on the state of the general economy $Y$. Thus we evaluate the probability of default conditionally on the realization of the systematic factor $y$. This can be interpreted as assuming various scenarios for the economy, determining the probability of default under each scenario, and then weighting each scenario by its likelihood. The conditional probability of default is

$$
\mathbb{P}(Z_i = 1|Y = y) = \mathbb{P}(R_i \leq \Phi^{-1}(p_i)|Y = y)
$$

$$
\mathbb{P}(Z_i = 1|Y = y) = \mathbb{P}\left(\sqrt{\rho_i Y} + \sqrt{1 - \rho_i}\varepsilon_i \leq \Phi^{-1}(p_i)|Y = y\right)
$$

$$
\mathbb{P}(Z_i = 1|Y = y) = \mathbb{P}\left(\varepsilon_i \leq \frac{\Phi^{-1}(p_i) - \sqrt{\rho_i Y}}{\sqrt{1 - \rho_i}}\right)
$$

Finally, knowing that the idiosyncratic risk factor follows a normal standard distribution we get the following formula:

$$
\mathbb{P}(Z_i = 1|Y = y) = \Phi\left(\frac{\Phi^{-1}(p_i) - \sqrt{\rho_i Y}}{\sqrt{1 - \rho_i}}\right)
$$

Consequently, we can affirm that Conditional PD formula is nothing else than a function of PD and the state of the economy $y$.

We can describe the Loss of a portfolio as follows

$$
L = \sum EAD_i LGD_i Z_i
$$

The ASRF framework assumes an infinitely granular portfolio and the existence of only one systematic risk factor. With these two assumptions fulfilled the $\alpha$-quantile of the loss $q_\alpha(L)$ is almost surely equal:

$$
q_\alpha(L) = \sum EAD_i LGD_i \Phi\left(\frac{\Phi^{-1}(p_i) + \sqrt{\rho_i \Phi^{-1}(\alpha)}}{\sqrt{1 - \rho_i}}\right)
$$

Formally, we have the worst-case scenario when the systematic factor takes the worst magnitude. $Y$ is a Standard normal variable, so this magnitude is given by $-\Phi^{-1}(\alpha)$ with some confidence level $\alpha$.

Under this worst-case scenario we have the most serious loss and the capital requirement is then given by:
Capital Requirement \((PD, \alpha, \rho, EAD, LGD) = \text{Worst Loss} - \text{Expected Loss}\)

\[
C_\alpha(L) = q_\alpha(L) - \mathbb{E}(L) = \sum EAD_iLGD_i \left( \phi \left( \frac{\Phi^{-1}(p_i) + \sqrt{\rho_i} \Phi^{-1}(\alpha)}{\sqrt{1 - \rho_i}} \right) - p_i \right)
\]

\[
K = LGD_i \left( \phi \left( \frac{\Phi^{-1}(p_i) + \sqrt{\rho_i} \Phi^{-1}(\alpha)}{\sqrt{1 - \rho_i}} \right) - p_i \right)
\]

If we insert the confidence level \(\alpha = 99,9\%\) we find the regulatory formula for capital charge coefficient with one year maturity. And risk weight function is directly proportional to the capital ratio.
2. Risk weight functions limits

Now that we explained how risk weight functions are constructed, we will continue to the core part of our article, which introduces the different limits appearing in the risk weight functions, their calibrations and impacts on the economic capital required.

In the following analysis, when not specified, we use the BCBS assumptions of the foundation IRB approach, namely, LGD = 45% and $\alpha = 99.90\%$.

2.1. Dependence on PD, LGD

Since the regulatory capital charge formula is a function of PD and LGD, one can’t help but wonder to what extent does the capital charge depend on these significant input parameters?

In order to respond to this essential question, we analyze the capital charge coefficient assuming no maturity adjustments function. The maturity adjustments function will be discussed further. First, as function of PD with a fixed LGD at 45%.

![Graph: Regulatory capital charge (K) as function of PD](source: Global Research & Analytics analysis)

In general, the capital charge coefficient increases with PD until the inflexion point, which then causes the capital charge to decrease. This could be explained by the fact that once that inflexion point is reached, losses are no longer absorbed by UL but by EL, thus lowering the economic capital required.
As a function of LGD, the capital charge coefficient is directly proportional to it. The figure below shows it with a fixed PD of 0.1%.

![Regulatory capital charge (K) as function of LGD](image)

*Source: Global Research & Analytics analysis*

Finally, we study the impact of both LGD and PD on capital ratio.

![Regulatory capital charge (K) as function of PD and LGD](image)

*Source: Global Research & Analytics analysis*

Considering our previous points, the regulatory capital charge evolves as expected. Increasing with LGD and the presence of an inflexion point for PD. Practical cases are subject to low
PD, so the linear dependence on LGD will principally impact capital ratio strongly than PD. It’s shown by the dark blue areas in the figure below.

![Capital ratio (K) for low PD](image)

*Source: Global Research & Analytics analysis*

### 2.2. Supervisory estimates of asset correlations

In the formula, $\rho$ represents the correlation coefficient. The correlations can be described as the dependence of the asset value of a borrower on the general state of the economy. Different asset classes show different degrees of dependency on the overall economy, so it’s necessary to adapt the correlation coefficient to these classes.

The asset correlation function is as follows:

$$
\rho = 0.12 \left( \frac{1 - e^{-50PD}}{1 - e^{-50}} \right) + 0.24 \left[ 1 - \left( \frac{1 - e^{-50PD}}{1 - e^{-50}} \right) \right]
$$

The asset correlation function is built on two limit correlations of 12% and 24% for very high and low PDs (100% and 0%, respectively).
However, this formula is incomplete. For Corporate exposures, it has been shown that asset correlations increase with firm size\(^3\). The higher the size of a firm, the higher is its correlation with the state of the economy. So for SME it’s necessary to adjust the asset correlation function to highlight their idiosyncratic risk factor. The asset correlation function is then:

\[
\rho = 0.12 \left( \frac{1 - e^{-0.5PD}}{1 - e^{-0.5}} \right) + 0.24 \left[ 1 - \left( \frac{1 - e^{-0.5PD}}{1 - e^{-0.5}} \right) \right] + \text{Size adjustment function}
\]

With:

\[
\text{Size adjustment function} = \begin{cases} 
0 & \text{for } S > 50 \\
-0.04 \left( 1 - \frac{S - 5}{45} \right) & \text{for } S \in [5,50] \\
-0.04 & \text{for } S < 5
\end{cases}
\]

Where \( S \) is the firm size.

This size adjustment concerns only borrowers with annual sales below €50 MN.

The following figure shows the regulatory Capital Charge for different firm sizes without maturity adjustment.

\(^3\) The size here is measured by annual sales.
This figure shows that at low PD, capital charge coefficient increases with it. Furthermore, we observe that at same PD for different firm sizes the capital charge ratio is correctly distributed. Formally, a small firm will have a lesser or equal capital ratio than a larger one.

At this point, there is a belief amongst specialists and some central bankers that the Basel methodology unfairly penalises SMEs. The discussion revolves around rules set by the Basel Committee in 2005 concerning correlations.

These rules, unchanged since then, cause an allocation of capital to SMEs that is much higher than the actual loss experience incurred by the SME sector historically and up to the present day. The work carried out by Mr. Dietsch and J. Petey in France is presented hereinafter. It shows the real discrepancy between regulatory correlations and those actually observed on conventional SME portfolio:
In the chart below, we show the risk weighted asset calculation for SMEs based on the Basel rules, and then on actual experience in the sector. The result is a significantly reduced allocation of capital if actual experience is used instead of the existing rules.

Finally, the presence of bounds can be explained by a lack of historical data for large and small firm, thus calibration is harder to quantify. Hence these bounds represent the global dependency of small and large firm on the overall economy.

Another asset correlation function has to be built for retail exposures. Indeed retail portfolio is subject to low correlation because the default of retail customers tends to be more idiosyncratic and less dependent on the economic cycle than corporate default.
Residential Mortgages: $\rho = 0.15$

Qualifying Revolving Retail Exposures: $\rho = 0.04$

Other Retail Exposures: $\rho = 0.03 \left( \frac{1 - e^{-35PD}}{1 - e^{-35}} \right) + 0.16 \left[ 1 - \left( \frac{1 - e^{-35PD}}{1 - e^{-35}} \right) \right]$

This calibration of the asset correlation function has been made over the analysis of historical banks economic capital data. This new function reflects the relatively high and constant correlation for residential mortgage exposures, the relatively low and constant correlation for revolving retail exposures, and, similarly to corporate borrowers, a PD-dependent correlation in the other retail case.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{example.png}
\caption{Regulatory capital ratio (K) for retail exposures}
\end{figure}

Source: Global Research & Analytics analysis

Finally, the impact of correlation on the capital ratio will be significant for high PD exposure.

2.3. From LGD to downturn LGD

Since we apply the ASRF model, parameters have to be conditioned by the overall state of the economy.

Though LGD plays a great role in the capital requirement function, Basel II does not provide an explicit function that transforms average LGDs into conditional LGDs (dependent on the systematic factor).
According to the BCBS, there could be two approaches to derive “downturn” LGDs:

- Using a mapping function like the one used for PDs that would derive downturn LGDs from average LGDs.
- Based on the internal valuations of LGDs during the hostile circumstances to compute “downturn” LGDs.

Since the bank practices concerning the LGD quantification are so sophisticated, the BCBS was forced to affirm that it is unsuitable to use a single supervisory LGD mapping function.

In addition, the BCBS decided that Advanced IRB banks have to estimate their own “downturn” LGDs with the obligation that those “downturn” LGDs during the recession circumstances must be superior to LGDs during normal circumstances.

In the previous regulatory formula, only the average LGD was used. Capital ratio was directly proportional to it.

\[ UL_{previous} = LGD_{average} \times (Cond PD - PD) \]

Finally, the “downturn” LGD was introduced in the regulatory formula substituting the previous average LGD, in order to keep the proportionality.

\[ UL_{current} = LGD_{downturn} \times (Cond PD - PD) \]

Knowing the “downturn” LGD is superior to the average version, we can claim that the new regulatory formula is conservative. However an intuitive approach could use both LGDs in the formula. In order to face the “downturn” LGD to the conditional PD and the average LGD to the average PD:

\[ UL_{intuitive} = LGD_{downturn} \times Cond PD - LGD_{average} \times PD \]

Furthermore, the current version is beneficial to the bank since the current capital ratio is less than its intuitive version. It’s shown below.

\[ UL_{current} - UL_{intuitive} = LGD_{downturn}(Cond PD - PD) - (LGD_{downturn} Cond PD - LGD_{average}PD) \]

\[ UL_{current} - UL_{intuitive} = PD(LGD_{average} - LGD_{downturn}) < 0 \]
Because

\[ LGD_{\text{average}} < LGD_{\text{downturn}} \]

So

\[ UL_{\text{current}} < UL_{\text{intuitive}} \]

The figure below shows our different cases, with an average LGD at 35% and a downturn LGD at 45%.

Source: Global Research & Analytics analysis

At low PD, we notice a similar comportment between the current formula and the intuitive one. Moreover the current formula is easier to compute since it only depends on downturn LGD which could explain why this version was chosen by the Basel committee.

2.4. The maturity adjustment function

For two counterparties with the same degree of solvency, credit risk is not necessarily the same as everything here depends on the loan maturity. With the same degree of solvency, the higher is the loan maturity, the higher is the credit risk.

Since this maturity affects credit risk, it was necessary for the Basel committee to take it into account and adjust the capital charge formula.
According to the BCBS, there are 3 ways to consider the maturity adjustments:

The first one is considering the maturity adjustments as a consequence of increasing credit risk. The fact that credit risk increases with increasing maturity has to imply more capital charge (loan maturity effect).

The second one is considering the maturity adjustment as an anticipation of additional capital charge due to downgrades. Indeed, downgrades are more likely when it comes to long-term credits and hence the anticipated capital charge will be higher than for short-term credit (potential downgrades effect).

The third one is considering the maturity adjustments like a consequence of mark-to-market (MtM) valuation of credits. The expected losses are taken into account by investors when pricing the fair value of loans. Those expected losses are increasing with ascending PDs and decreasing with descending PDs. Therefore, the market value of loans decreases with increasing PDs and increases with decreasing PDs (marked to market losses effect).

For these reasons the BCBS integrated a maturity adjustments function to the basic regulatory formula to take into account maturity effects: loan maturity, potential downgrades and marked to market losses.

The maturity adjustments function is a function of both maturity and PD, and they are higher (in relative terms) for low PD than for high PD borrowers. The BCBS assumed a standard maturity of 2.5 years, so they constructed the function around this standard maturity. It’s presented as follows:

$$ \text{MA} = \frac{1 + (M - 2.5)b(PD)}{1 - 1.5b(PD)} $$

*With the smoothed maturity adjustment:

$$ b(PD) = (0.11852 - 0.05478 \log(PD))^2 $$

The figure below shows the maturity adjustment as a function of PD and maturity, reflecting the potential downgrade effect and the loan maturity effect.
It’s worth mentioning for loan maturity less than one year, the adjustment increases with increasing PD but is still lower than 1. Consequently, the capital charge is lower than the case without maturity adjustment. For maturity longer than one year, the adjustment decreases with increasing PD but is still greater than 1, and consequently the capital ratio is also greater than the case without maturity adjustment.

Considering the maturities adjustment curves, we can wonder how the capital ratio is impacted by the maturity adjustment. The figure below shows the capital ratio as a function of PD and maturity thus considering the maturity adjustment.
We observe the behaviour of the capital ratio curve remains the same whereas it's supposed to be proportional to the maturity adjustment coefficient. This can be explained because both the capital and maturity adjustment ratios depend on the PD. Thus the capital ratio curve maintains its state and is linearly dependent on the maturity only.
2.5. A confidence level at 99.9%

According to the BCBS, the benchmark for setting the confidence level is BBB+ firms that have a default probability fixed on average at 0.1%, which amounts to say that the probability of survival is set at 99.9%. In other words, the BCBS is confident on average at 99.9% that a BBB+ rated bank would survive in the time horizon of 1 year.

That’s why the confidence level is set at 99.9%. It means a bank can have losses that exceed its tier 1 and tier 2 capital on average once in a thousand years. Moreover it was also chosen to protect against estimation errors, which might inevitably occur from banks’ internal PD, LGD and EAD estimation, as well as other modeling uncertainties.

Even if a bank is below the BBB+ rating, it has to respect the 99.9% confidence level.

Nowadays most of the banks are above the BBB+ rating so currently the fixed confidence level is permissive, although this wasn’t always the case during the implementation of Basel agreements.

In the future, we can expect an increase in confidence in order to better match the actual economy and its evolution.

However, a decrease in confidence could be caused by a critical and massive downgrading of the banking sector.

The figure below shows capital ratio measurement for different confidence level.
Formally, economic capital increases with increasing confidence levels. The higher the confidence level; the higher the economic capital is and vice-versa.

2.6. The standard normal distribution in the ASRF framework

The statistical properties of the Normal Law allow the Basel Committee to provide a closed formula for the regulatory RW calculation. Such properties remain a strong hypothesis of the ASRF model (as it exists on the market risk approach). These are reminded hereafter:

Indeed, let $A_i$ be the value of a borrower assets, described by the process:

$$dA_i = \mu A_i dt + \sigma A_i dx$$

Where asset value at $T$ can be represented as:

$$\log A_i(T) = \log A_i + \mu T - \frac{1}{2} \sigma^2 T + \sigma \sqrt{T} X$$

And $X$ is a standard normal variable.

We explained in part one that an asset $i$ defaults if its value goes below a critical threshold $S_i$. Such as

$$p_i = \mathbb{P}(A_i \leq C_i) = \mathbb{P}(X < S_i) = \phi(S_i)$$
Where

\[ S_i = \frac{\log C_i - \log A_i - \mu T + \frac{1}{2} \sigma^2 T}{\sigma \sqrt{T}} \]

\( \phi(.) \) is a cumulative normal distribution function.

The variable \( X \) is standard normal, and can therefore be represented as

\[ X = \sqrt{\rho_i} Y + \sqrt{(1 - \rho_i)} \varepsilon_i = R_i \]

Where \( Y \) and \( \varepsilon_i \) are mutually independent standard normal variables, and are the risk factor presented in part one.

Hence it explain the standard normal distribution used in the ASRF framework.

2.7. Processing portfolios default

Once default is observed, loss distribution is bimodal. In the cases that appear, almost all or nothing is recovered. The figure below shows the loss distribution in case of default.

For defaulted portfolios PD is equal to 100%, and as a consequence, risk weight functions are not measurable. Hence expected loss and unexpected loss have to be recalculated together with LGD on defaulted exposures.
Expected losses for defaulted portfolios depend only on expected recuperations on defaulted exposures. This best estimation of expected loss is called BEEL. Specific provisions are here to cover the BEEL.

While default LGD is more severe than normal LGD, it’s supposed to estimate a downturn effect.

The measurement of unexpected losses relies on the difference between loss recovery and their best estimation. It’s estimated as follows:

\[
UL = \max(0; \ LGD_{default} - BEEL)
\]

- If \( BEEL \geq LGD_{default} \): there is no load in economic capital required (e.g. best estimation of expected loss is conservative)
- Else more economic capital is required to cover these unexpected losses (e.g. best estimation of expected loss is insufficient)

As a conclusion, in order to process defaulted portfolios, banks have to estimate their own BEEL, default LGD and specific provisions. Methodologies differ from banking establishment, thus the various ways to estimate these parameters will have different impacts on the solvency ratio.
Conclusion

After reviewing the funding principles behind Basel II and explaining the construction of risk weight functions, we exhibited input parameters of the function and detailed their limits, calibrations and impacts over the capital charge coefficient.

The capital ratio is mainly dependent on PD and LGD; when both of them vary at low levels, the capital ratio is stable. High variations will occur when there are higher levels of these parameters, especially LGD since the ratio is directly proportional to it.

Others parameters behind the Basel regulatory formula can explain a large number of adjustments made by financial institutions in the implementation of Basel II. In this sense, the assumptions and shortcuts decided in the regulatory RWA calibration need to be known and understood. In return, this would help banks refine their vision of economic capital and to better measure their actual risk (defaulted portfolio, financing cost of capital for SME, impact of systemic risk actually found in their own jurisdiction...).

To conclude, having an in-depth understanding of the theoretical foundations behind the Basel formula will allow for opportunities to better address weaknesses and to help to uncover unexpected risks of banks.
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